

# Stirling System Modeling for Space Nuclear Power Systems

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**Abstract** - A dynamic model of a high-power Stirling convertor has been developed for space nuclear power systems modeling. The model is based on the Component Test Power Converter (CTPC), a 12.5-kWe free-piston Stirling convertor. The model includes the fluid heat source, the Stirling convertor, output power and heat rejection. The Stirling convertor model includes the Stirling cycle thermodynamics, heat flow, mechanical mass-spring damper systems, and the linear alternator. The model was validated against test data.

Both nonlinear and linear versions of the model were developed. The linear version algebraically couples two separate linear dynamic models; one model of the Stirling cycle and one model of the thermal system, through the pressure factors.

Future possible uses of the Stirling system dynamic model are discussed. A pair of commercially available 1-kWe Stirling convertors is being purchased by NASA Glenn Research Center. The specifications of those convertors may eventually be incorporated into the dynamic model and analysis compared to the convertor test data. Subsequent potential testing could include integrating the convertors into a pumped liquid metal hot-end interface. This test would provide more data for comparison to the dynamic model analysis.

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# Outline

- Why dynamic system modeling?
- Component Test Power Convertor (CTPC)
- Nonlinear Stirling dynamic model
- Linear Stirling model
- Thermal system model
- Coupling linear Stirling model with the thermal system model
- Simulation results
- Future modeling efforts
- Conclusions

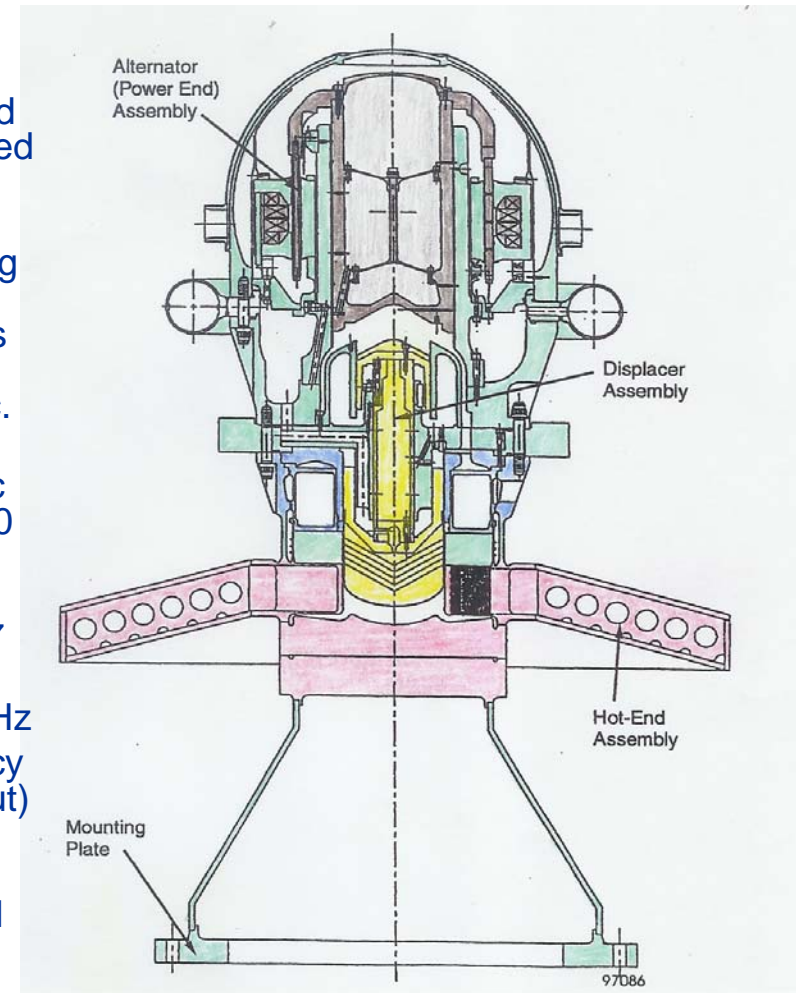
# Why dynamic system modeling?

- Dynamic models beneficial for
  - Trade studies
  - Evaluation of design options
  - Dynamic performance prediction
  - Failure effects analysis
  - Design optimization
  - Controls development
- Important for space nuclear power systems because of
  - Cost of prototypes
  - Complexity of the system

# Component Test Power Convertor (CTPC)

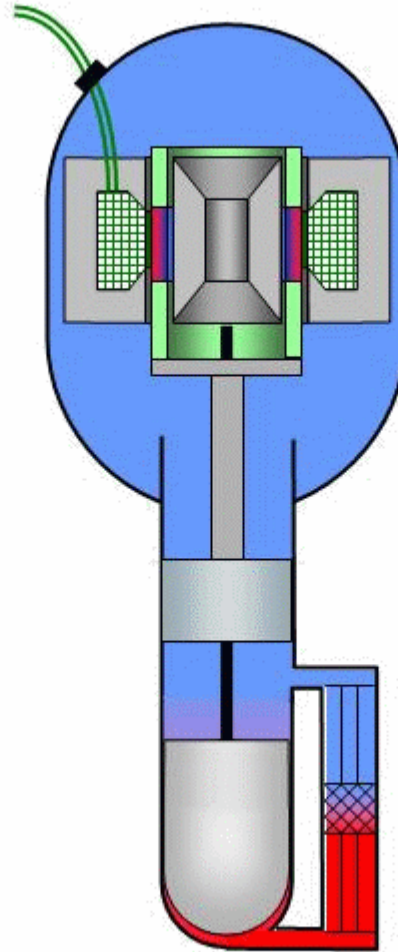


- CTPC is closest Stirling convertor in power level and design to what might be used as part of a space nuclear power system
- 12.5 kWe free-piston Stirling convertor designed, built, and tested in the late 1980s and early 1990s by Mechanical Technology Inc. (MTI)
- Heat in from radiant electric heaters or heat pipes at 800 K (527 °C); operated up to 1050 K (777 °C)
- Heat rejected at 400 K (127 °C)
- Operating frequency is 70 Hz
- Overall conversion efficiency (heat in to electric power out) of 22%
- Model can be upgraded as new designs are developed



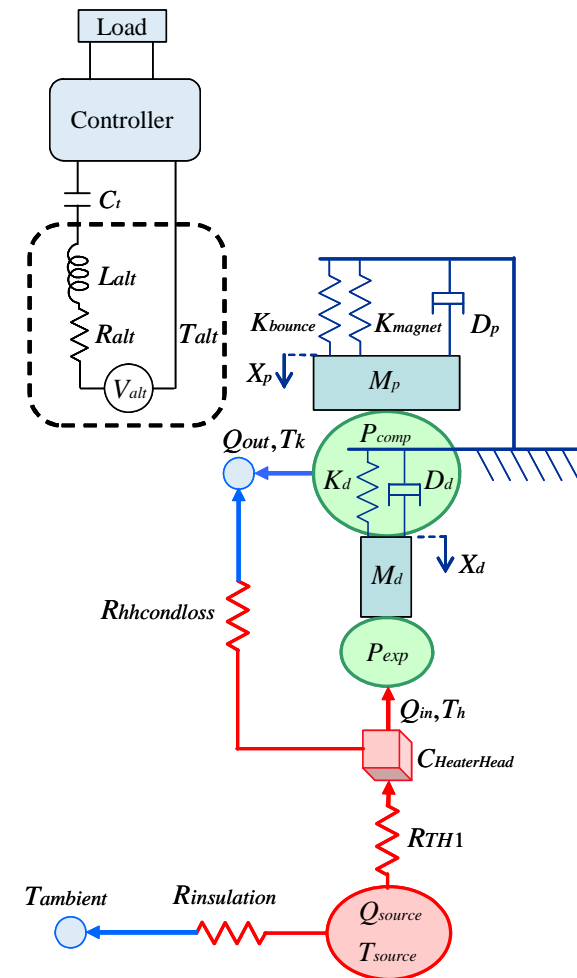
# Stirling cycle

- Dynamic model captures key parameters of free-piston Stirling convertor operation



# CTPC dynamic model

- Heat input  $Q_{source}$
- Stirling cycle thermodynamics modeled based on the Schmidt model (assumes isothermal Stirling cycle)
- Pressure forces, gas spring and damping forces act on the displacer and piston masses.
- Alternator damping force and magnet spring force
- Controller electrical dynamics



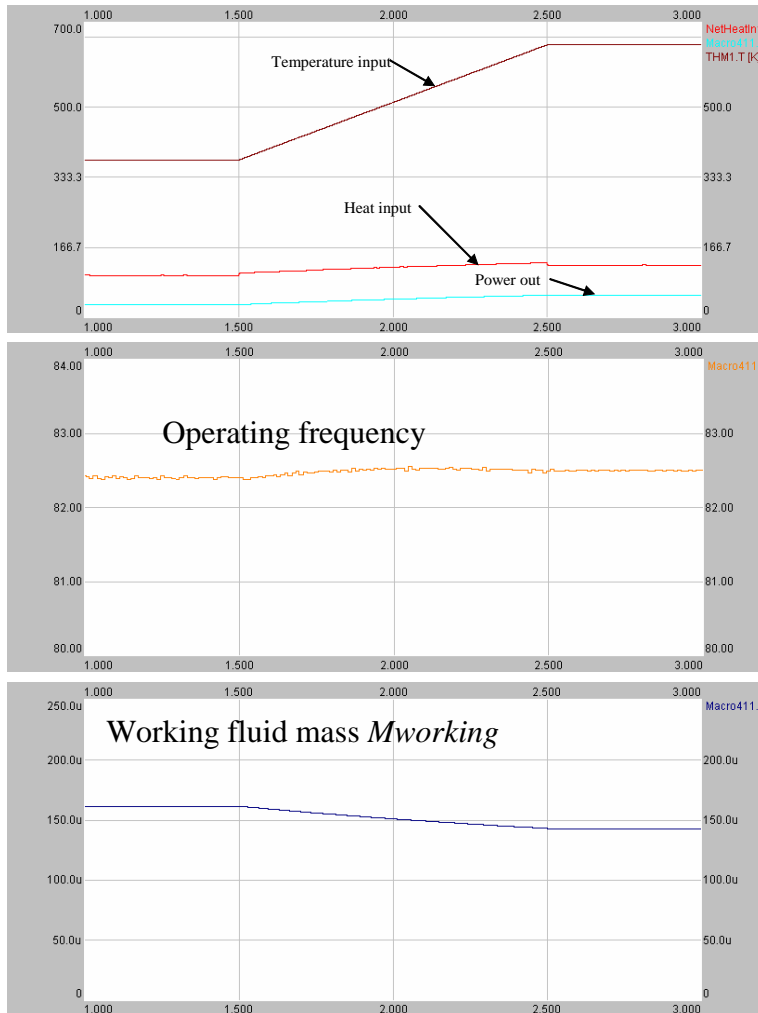
# SDM – nonlinear Stirling model

- NASA Glenn Research Center developed the Stirling convertor System Dynamic Model (SDM) for simulation and analysis
- Ansoft Simplorer software platform
- Includes many system nonlinearities:
  - Gas dynamics
  - Fluid flow and pressure drop
  - Nonlinear springs
  - Temperature effects
  - Mechanical limits
  - Electrical components
  - Controller
- Can be coupled with a Sage model of the thermodynamics to improve model fidelity.



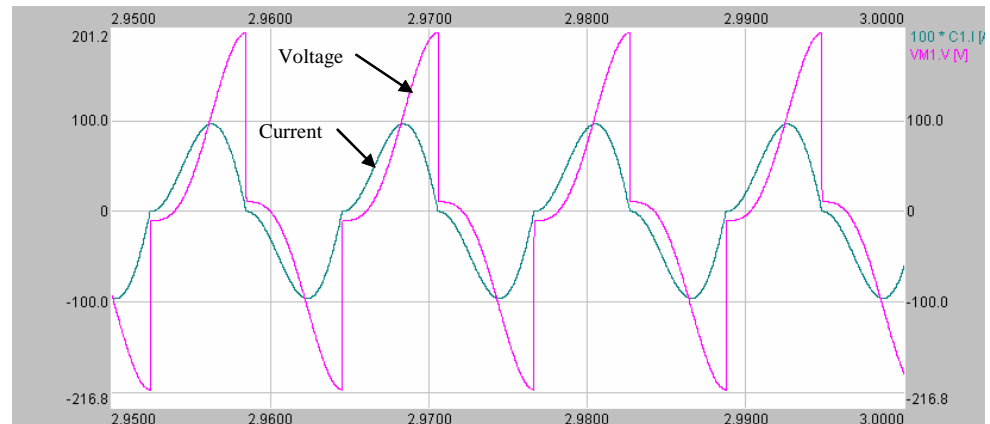
# SDM capability

## Thermal transient



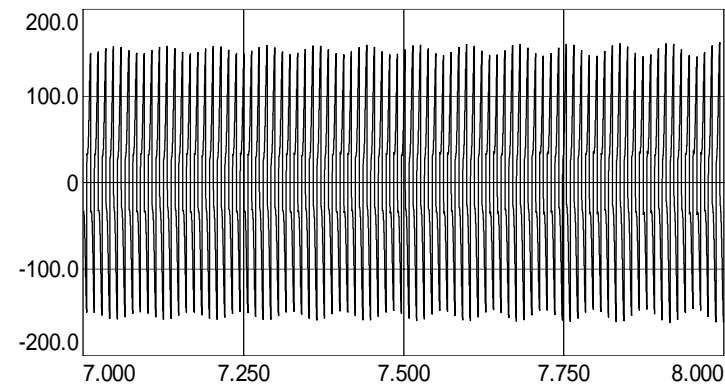
seconds

## Nonlinear current and voltage waveforms



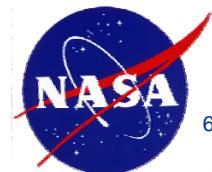
milliseconds

## voltage oscillation due to poorly tuned controller



**Sest, Inc.**

Glenn Research Center at Lewis Field



# Linear Stirling model

- Linear models useful
  - Easier to port model to other software platforms
  - Faster simulation times
  - Controller design and analysis
- Pressure factors characterize Stirling cycle operating point

$$\frac{dx_d}{dt} = \dot{x}_d$$

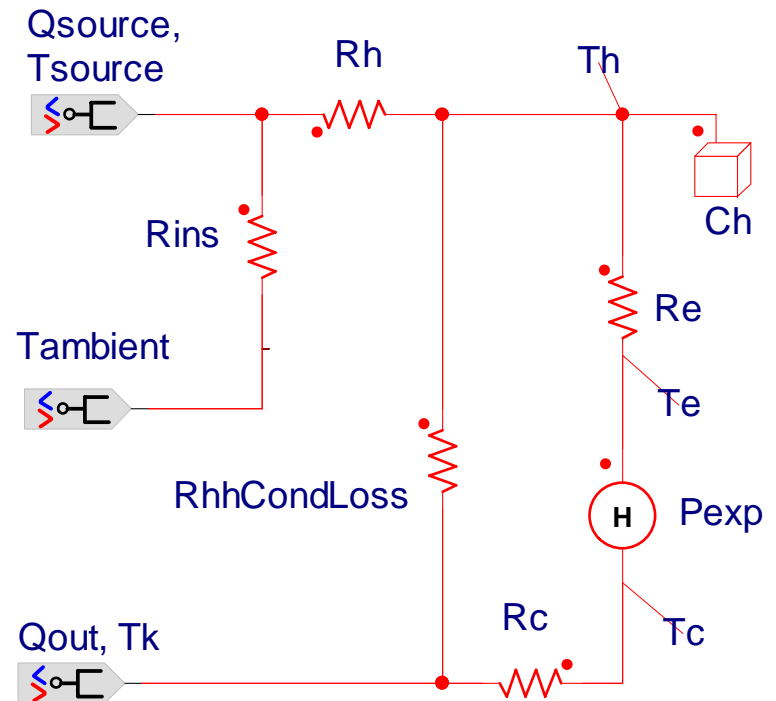
$$\frac{d\dot{x}_d}{dt} = -\left(K_d + A_r \frac{\partial P}{\partial x_d}\right) \frac{1}{M_d} x_d + \left(A_d \frac{\partial \Delta P}{\partial \dot{x}_d} - C_d\right) \frac{1}{M_d} \dot{x}_d - A_r \frac{\partial P}{\partial x_p} \frac{1}{M_d} x_p + \left(A_d \frac{\partial \Delta P}{\partial \dot{x}_p}\right) \frac{1}{M_d} \dot{x}_p$$

$$\frac{dx_p}{dt} = \dot{x}_p$$

$$\frac{d\dot{x}_p}{dt} = -A_p \frac{\partial P}{\partial x_d} \frac{1}{M_p} x_d - \left(K_p + A_p \frac{\partial P}{\partial x_p}\right) \frac{1}{M_p} x_p - \frac{C_p}{M_p} \dot{x}_p + N \frac{d\Phi}{dx_p} \frac{1}{\eta_{mag}} \frac{1}{M_p} I_{alt}$$

# Thermal system model

- Thermal resistances model temperature drops in system
- Thermal inertia models heat capacitance of heater head
- Heat flow into Stirling cycle determined by thermal dynamics



# Coupling linear Stirling model and thermal system model

- Stirling cycle response time is on the order of milliseconds
- Thermal system response time is on the order of seconds or minutes
- Algebraically couple systems through pressure factors
- Pressure factors calculated based on temperatures and heat flows
- Pressure factors define dynamics of Stirling cycle

$$\frac{\partial P}{\partial x_p} = \frac{A_p M_w R_{gas}}{(Q_h R_c + T_k)} \cdot \left[ \frac{V_{co} - A_p x_p + (A_d - A_{rod}) X_d}{Q_h R_c + T_k} + \frac{V_k}{T_k} + \frac{V_r \ln(T_h / T_k)}{T_h - T_k} + \frac{V_h}{T_h} + \frac{V_{eo} - A_d x_d}{T_h - Q_h R_e} \right]^{-2}$$

$$\frac{\partial P}{\partial x_d} = -M_w R_{gas} \left[ \frac{(A_d - A_{rod})}{Q_h R_c + T_k} - \frac{A_d}{(T_h - Q_h R_e)} \right] \cdot \left[ \frac{V_{co} - A_p x_p + (A_d - A_{rod}) X_d}{Q_h R_c + T_k} + \frac{V_k}{T_k} + \frac{V_r \ln(T_h / T_k)}{T_h - T_k} + \frac{V_h}{T_h} + \frac{V_{eo} - A_d x_d}{T_h - Q_h R_e} \right]^{-2}$$

- Further details on coupling Stirling linear model with thermal system model can be found in Regan and Lewandowski "Development of a Linear Stirling System Model with Varying Heat Inputs," IECEC 2007

# Simulation results

- Compare nonlinear model with test data
- Compare linear model with nonlinear model

Parameter	units	800 K test data	SDM nonlinear model		Linearized model	
			Value	error vs. test data	Value	error vs. nonlinear model
Power out	W	12,780	12,891	0.9%	12,863	-0.2%
current	Arms		48.09		48.04	-0.1%
voltage	Vrms		401.0		402.3	0.3%
frequency	Hz	67.45	67.48	0.0%	66.77	-1.1%
XDamplitude	m	0.01480	0.01266	-14.4%	0.01294	2.2%
XPamplitude	m	0.01344	0.01363	1.4%	0.01372	0.7%
displacer phase angle	deg	70.83	68.87	-2.0	81.53	12.7°
mean pressure	Pa	15,000,000	15,020,114	0.1%	n/a	
pressure amplitude	Pa	1,600,000	1,247,179	-22.1%	n/a	
pressure phase angle	deg	-12.48	-15.79	-3.3	n/a	
alternator efficiency	%	87.84%	90.62%	3.2%	n/a	
Th	K	800	799.7	0.0%	799.7	0.0%
Te	K	776	779.7	0.5%	779.6	0.0%
Tc	K	418.5	415.4	-0.8%	415.5	0.0%
Tk	K	400	400.0	0.0%	400.0	0.0%

# Future modeling efforts

- CTPC represents state-of-the-art from late 1980's
- NASA GRC is procuring dual-opposed 1-kW convertors to investigate heater head concepts
- Create nonlinear and linear models of 1-kW convertors
- Test dynamic response to changes in load, amplitude, or frequency.
- Eventually couple convertors to a liquid metal heat transfer loop at MSFC
- Incorporate Stirling model into a larger system model including reactor, heat pipes, radiators, controller, etc.

# Conclusions

- Dynamic model of high power Stirling convertor developed
- Linear model with thermal system developed that can be coupled with other system models for end-to-end system simulation
- Model compares favorably with test data
- Newer Stirling convertor hardware is being procured for further testing and analysis in support of Fission Surface Power

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